Triples by The CQ Elves

Solution

- Remember that within a triple the entries are in ascending order apart from the string components in III and VI.
- Clue I has only one set **T=361**, **U=529**, **S=784**. From this we can deduce that G in clue III ends in 8.
- Clue VII, the cube at s' must be 512 so **s = 215**. One of the triangular numbers must be less than 512 and the other greater. The digits remaining are 3, 4, 6, 7, 8 and 9. For the one less than 512 we can have 378 or 496. If we choose the former then there is no fit for the larger one using the digits 3, 7 and 8. So, **E=378** and **J=946**.
- Clue IV **c=738** the only possibility.
- In clue VI A, Q and r are all odd so k is odd. We can go further in that we know the terminal digits of A, Q and r which are 7, 1 and 1 respectively. We have $k^3 + 1^3 + 7^3$ ending in 1 thus k^3 must end in 7 so k ends in 3.
- The digit sums in clue II fall into two sets (168),(249),(357) or (159),(267),(348). We can eliminate the first set from clue VI in that A cannot contain 1, 3 or 4 since k is 4?3 and r ends in 1. So, **A=627** with D either 159 or 195 and G 348 or 438. However, by using clue IV we can rule out D as 159 so **D=195**. Hence, n is clue IV can be 264 or 624.
- The product in clue III is either equal to 195627348 or 195627438. Factorising these gives $2^2 \times 3^2 \times 7 \times 47 \times 83 \times 199$ and $2 \times 3^2 \times 733 \times 14827$. We can rule out the latter in that it has a 5-digit prime. Thus, **G=348** (this completes clue II) and **b=254**. The 199 factor in III must end in 1, 3 or 6 which means it must be multiplied by 9, 7 or 4 respectively. Only the latter gives a 3-digit result. So, **q=796** and is the largest. From the remaining factors we can find p and r which are from $3^2 \times 47$ and 7×83 which yield **p=423**, **r=581** which gives **Q=981** by default and hence the value of *n* in clue XI which gives the triple 327, 654, 981. Also, **M=475** by default.
- This allows the value of n in clue IV to be assigned as **n=264** as P cannot contain two 2s.
- The square in clue IX is 729 which is M + b.
- From clue VII we know that N is a square starting with 2 and can be 256 or 289. This gives k as 463 or 493 in clue VI. It cannot be the former as A contains a 6. Hence, **k=493** and we get $493^3 + 581^3 + 627^3 = 562437981$ which gives **P=562** and **m=437** with Q already known to be 981. This assigns **N=289** as the square in clue VII.
- Clue V is ?8? + 4?2 = ??9. This forces the last digit in e to be 7 with 1, 3, 5 and 6 left. K cannot be 412 or 462 as that results in duplicate digits. If it's 432 then that just leaves 5 and 6 and no valid sum can be made. Thus, **M=452**, **e=187** and **g=639**. This yields **j=935** by default.
- The middle term in the clue XI triple is 654 and is 8?2 −?1?. This forces B to terminate in 8 and the solution is **t=872** and **B=218.** This forces u to end in 5 which it can as it's a triangular number from clue VII. In fact, u must start with at

least a 4 and this gives 435, 465 and 595 as the only possibilities. We can rule out the latter as that has repeated digits for u and the former has repeated digits for R. Thus, **u=465** with **R=736**. This allows the prime in clue VII to be found and so **a=317**.

- Clue XII f can be 824 or 842. If we choose the former then F, a triangular number from clue IX, would be 231. However, the square in that triple is 729 which duplicates a 2. This gives **f=842** and **F=435**. This forces the second triangular number in IX to be 861 which gives **H=572** and **d=275** by default.
- We can calculate h in clue XI and it is **h=126**.
- We can now calculate the middle term in clue X which is 598 and finally the last term 6?1 which is a multiple of 23 so **C=621**.

a 3	A 6	b 2	c 7	B d 2	e 1	f 8	C g 6	2	h 1
D 1	j 9	5	E 3	7	8	F 4	3	5	2
7	G 3	4	8	H 5	7	2	J 9	k 4	6
K m 4	5	n 2	M p 4	q 7	r 5	N s 2	t 8	9	u 4
3	P 5	6	2	Q 9	8	1	R 7	3	6
S 7	8	4	T 3	6	1	U 5	2	9	5

A	627	K	452	a	317	k	493
B	218	M	475	b	254	m	437
C	621	N	289	c	738	n	264
D	195	P	562	d	275	p	423
E	378	Q	981	e	187	q	796
F	435	R	736	f	842	r	581
G	348	S	784	g	639	s	215
H	572	T	361	h	126	t	872
J	946	U	529	j	935	u	465

I	squares	361, 529, 784	T, U, S
II	digit sums = 15	195, 348, 627	D, G, A
III	product	423 x 581 x 796 = 195627348	p x r x q = D_A_G
IV	divisible by their digit sum	195, 264, 738	D, n, c
V	sum	187 + 452 = 639	e + K = g
VI	sum of cubes	$493^3 + 581^3 + 627^3 = 562437981$	$k^3 + r^3 + A^3$ = P_m_Q
VII	square, prime, triangular	289, 317, 465	N, a, u
VIII	triangular, cube, triangular	378, 512, 946	E, s', J
IX	triangular, square, triangular	435, 729, 861	F, M + b, H + N
X	multiple of 23	437, 598, 621	m, d + h + j – c, C
XI	$n : 2n : 3n$	327, 654, 981	h + u – n, t – B, Q
XII	set	195, 736, 842	D, R, f