# Crossnumber 

Puzzles

$$
15 \times 3 \times 5
$$

|  |  |  |
| :---: | :---: | :---: |
| , |  |  |
| - |  |  |

> By Zag

## Contents

Introduction ..... 3
Puzzles ..... 4
Helpful Hints ..... 19
Solutions ..... 20
Useful Numbers ..... 22

## Introduction

This booklet serves as a supplement to the regular puzzle offerings in Crossnumbers Quarterly. One aim of that magazine, besides providing some of the best crossnumber puzzles around, is to encourage new compilers into the field. This collection of 15 puzzles is intended as a learning resource or inspiration for would-be compilers and can be accessed on the www.crossnumbersquarterly.com website. The booklet also serves as an introduction to crossnumber puzzles for any interested solver wishing to check out what Crossnumbers Quarterly has to offer.

The puzzles are all based on a $3 \times 5$ grid, showing what can be achieved, even with a restricted format. They employ the most common types of number: prime, square, triangular, Fibonacci and occasionally puzzle specific palettes (tables of useful number types are attached at the back). The puzzles provide a range of difficulty which broadly increases as you work your way through the 15 examples, building up to a severe challenge at the end. If you need help getting started hints are provided immediately after the puzzles.

Would be compilers (or those just interested in the process) are encouraged to note the different approaches and devices that have been employed. Observe how the entry point for the puzzle is constructed. What part, if any, does the grid layout play? How much choice is maintained during the puzzle and is the level of challenge maintained? What would you have done differently? What factors determine the level of difficulty?

With your first attempt at compiling keep it small and simple. The important thing is to get a feel for how the elements come together. Use the examples here or any others you can find as inspiration for the type of puzzle you want to construct. The key consideration is be fair to the solver. Instructions must be clear and unambiguous. Execution should avoid overcomplex calculation or excessive logical pathways. The challenge level should be appropriate for the intended solvers and ideally even throughout. There should be no doubt in the solvers mind that he has completed the puzzle successfully. Most importantly make sure any puzzle has been thoroughly checked before releasing it into the world. In particular, ensure the clues are consistent with the solution and there have been no misprints or omissions. Check the completed grid obeys the preamble and does not include inadvertent duplicates or excluded numbers. Have an independent solver tackle the puzzle to ensure it is uniquely solvable with the intended solution.

As you gain experience you will naturally find yourself increasing the elegance, originality and interest of your puzzles. Good luck with your efforts and enjoy these puzzles.

## Zag

## 1. Going Up

Answers are distinct, none starting with a zero.


Across
1 strictly increasing digits
4 prime divisor(1ac)
5 digit sum(1ac)
6 strictly increasing digits

## Down

1 Fibonacci
2 square
3 Fibonacci

## 2. Prime Digit Sums

Each answer has a different one of the first 9 primes as its digit sum. No answer starts with a zero. There are just 4 clues.


## Across

1 square root of 5 dn
7 divisible by its digit sum
8 digit sum of 1ac

## Down

2 reverse of square

## 3. Three Squares

This puzzle features three squares. Answers are distinct, none starting with a zero.

Across
$\mathbf{1}$ square
$\mathbf{3}$ reverse( 6 ac )
$\mathbf{4}$ square
$\mathbf{6}$ see 3 ac
$\mathbf{7}$ square

## Down

1 multiple(6ac)
2 prime
3 prime reverse(2dn)
5 multiple(3ac)
7 square

## 4. Triple Triangulars

This puzzle features three triangular numbers. As usual no answer starts with a zero and answers are distinct.

Across
1 prime
3 6ac-5dn
4 triangular number
6 divisor 4ac
7 anagram (1ac)

## Down

1 square root (7ac)

## 5. Fair and Square

Answers are distinct, none starting with a zero.


## Across

2 square \& multiple of 3dn
5 see $4 d n$
6 see 1dn
7 square

## Down

1 square of 6ac
2 square
3 anagram of a square
4 square of 5ac
6 square

## 6. Egofrequent

Each digit appears with its own frequency. Answers are distinct, none starting with a zero.

Across
Down
1 divisible by its digit sum
3 square
5 multiple of product of frequencies present
6 divisor(7ac)
1 prime
2 divisible by its digit sum
3 square
7 see 6ac

## 7. Palindromic Pair

Answers are distinct, none starting with a zero.


## Across

2 palindrome \& multiple(5ac)
5 prime
6 prime reverse(5ac)
7 palindrome \& multiple(6ac)

## Down

1 prime
3 square
4 prime \& (triangular-1dn)

## 8. Chessboard

Shaded squares have digits of one parity and plain squares have the opposite parity. Answers are distinct, none starting with a zero.


## Across

1 square
3 square
7 square
8 triangular
9 triangular

## Down

2 square
4 triangular
5 another answer - 6dn
6 see 5 dn

## 9. Fibonacci Duo

The duo of the title are 1ac \& 7ac. Answers are distinct, none starting with a zero.


## Across

1 Fibonacci
3 multiple(6ac)
4 multiple of 3dn digit sum

6 triangular-3ac
7 Fibonacci

Down
1 divisor(1ac)
2 palindrome \& multiple of another answer

3 palindrome forming a prime when added to 2 dn

5 divisor(7ac)

## 10. Mutual Replacement

Answers that include a 3 have any such 3 replaced by a 5 .
Similarly, 5's are replaced by 3 's. Clue references are to the entries so formed. Entries are distinct, none starting with a zero.


## Across

1 prime
4 multiple(square root(1ac))
6 digit product(6dn)
7 equals 7ac
8 multiple(square root( 9 ac ))

## Down

2 square
3 multiple(4ac)
5 prime
6 prime
7 square

9 triangular

## 11. Around the Square

The title refers to 5 ac, which is a square. Answers are distinct, none starting with a zero.


## Across

1 divisor(1dn)
2 prime
$52 \mathrm{ac}+6 \mathrm{ac}$
6 prime
7 divisor(4dn)

Down
1 not a multiple of 7 ac
3 square
$41 d n+6 a c-2 a c$
5 square

## 12. Unusual Friends

A friendly number is divisible by the sum of its digits. In this puzzle the answers are numbers that are divisible by the sum of their digits plus 1 , for example 231 which is divisible by 7 .

Answers are distinct, none starting with a zero and, apart from $2 \mathrm{ac} \& 5 \mathrm{ac}$, all are triangular.


## 13. Pandigital Series

The five 2-digit answers use the ten digits 0-9 to form an arithmetic series. Answers are unique, none starting with a zero.

A friendly number is divisible by the sum of its digits.


## Across

$6 \quad 2 \mathrm{dn}+4 \mathrm{dn}$

## Down

1 triangular
2 friendly
4 prime
5 triangular

## 14. Singles and Doubles

All the digits from 0 to 9 appear, with 2 different digits appearing in each column. Answers are distinct, none starting with a zero.


Across
1 see 4ac
4/7 one of them divides (8ac-1ac)

8 see 4ac

## Down

1 square
2 triangular total of 15 puzzle digits
3 composite
5 Fibonacci
6 triangular

## 15. Root Problem

This puzzle involves two triplets. The Pythagorean triplet $1 \mathrm{dn}, 4 \mathrm{dn}, 5 \mathrm{ac}$ has no common factor.

As usual no answer starts with a zero


## Across

1 such that $\sqrt{ } 1 \mathrm{ac}=\sqrt{ } 2 \mathrm{dn}+\sqrt{ } 7 \mathrm{ac}$
3 square
5 such that $5 \mathrm{ac}^{2}=1 \mathrm{dn}^{2}+4 \mathrm{dn}^{2}$
6 square
7 less than $2 d n$

## Down

1 greater than $4 d n$
2 see 1ac
3 reverse (5dn)
4 see 5ac
5 see $3 d n$

## Helpful Hints

## 1. Going Up

3dn choice is limited by its start and end digits.

## 2. Prime Digit Sums

Determine 3-digit primes with prime digit sums.

## 3. Three Squares

Needs of $2 \mathrm{dn}, 3 \mathrm{dn}$ uniquely determine middle digit of 1 ac .4 ac is palindrome.

## 4. Triple Triangulars

Lead digit of 1 dn occurs in its square (from 7ac clue).
5. Fair and Square

Consider lead digit of 5ac.
6. Egofrequent

0 cannot appear. Digit selection must allow prime, 2-digit square, 3-digit square \& triangular.

## 7. Palindromic Pair

Palindrome pqqp $=11^{*}\left(91^{*} \mathrm{p}+10^{*} \mathrm{q}\right)$. Use this and limited choice of $5 \mathrm{ac}, 6 \mathrm{ac}>11$.
8. Chessboard

Consider 7ac with alternating parities and their possible 4dn\&9ac.

## 9. Fibonacci Duo

Find possible $1 \mathrm{ac}, 1 \mathrm{dn}$ and $7 \mathrm{ac}, 5 \mathrm{dn}$ then apply fact that 2 dn a palindrome.
10. Mutual Replacement

6 ac is impossible without a lead digit replacement of 3 by 5 .

## 11. Around the Square

5ac starts at least $2.5 \mathrm{ac}, 3 \mathrm{dn}, 5 \mathrm{dn}$ must be such that $2 \mathrm{ac}, 6 \mathrm{ac}$ middle digits consistent with 5 ac .

## 12. Unusual Friends

Establish unusual triangular number palette. Only two 2-digit candidates, eight 3-digit.

## 13. Pandigital Series

Find the 4 possible pandigital series. Triangular endings cannot be 2,4,7,9.

## 14. Singles and Doubles

Determine range of 2 dn for different doubles. Only one triangular is consistent with double digits for that total.

## 15. Root Problem

6ac end digit is same as 3ac lead digit and 5ac is palindrome consistent with 4 dn being lowest member of triplet. The $1 \mathrm{ac}, 2 \mathrm{dn}, 7 \mathrm{ac}$ trio has the general solution:

$$
\mathrm{p} *(\mathrm{q}+\mathrm{r})^{2}, \mathrm{p} * \mathrm{q}^{2}, \mathrm{p} * \mathrm{r}^{2}
$$

## 1. Going Up

| 2 | 4 | 6 | 7 | 9 |
| :--- | :--- | :--- | :--- | :--- |
| 43 | 7 | 2 | 2 | 8 |
| 6 | 4 | 5 | 6 | 7 |

3. Three Squares

| ${ }^{1} 6$ | 2 | 7 | 6 | 3 |
| ---: | ---: | ---: | ---: | ---: |
| 3 | 2 |  |  |  |
| 9 | 4 | 8 | 4 | 9 |
| ${ }^{6} 2$ | 3 | 5 | 7 | 6 |

5. Fair and Square

| ${ }^{1} 7$ | ${ }^{2} 3$ |  | 3 | ${ }^{4} 6$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{5} 2$ | 6 | 1 | ${ }^{6} 2$ | 7 |
| 9 | 0 | 2 | 5 | 6 |

7. Palindromic Pair

| 1 | ${ }^{2} 7$ |  | 2 | ${ }^{4} 7$ |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{5} 7$ | 3 |  | ${ }^{6} 3$ | 7 |
| 3 | 6 | 6 | 63 | 3 |

9. Fibonacci Duo

| ${ }^{1} 6$ | 1 | 1 | 0 | 3 |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 8 | 4 |  |  |
| 1 | 4 | 4 | 3 | 7 |

11. Around the Square

|  | 9 | 2 | ${ }^{3} 3$ | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | 5 | 7 | 6 | 2 |
| ${ }^{6} 4$ | 4 | 3 | 5 | 5 |

13. Pandigital Series

14. Root Problem

| 8 |  | $\left.{ }^{2} 2\right]^{3}$ | ${ }^{3} 6^{4} 4$ |
| :---: | :---: | :---: | :---: |
| 5 | 9 |  | 92 |
|  | $6{ }^{7}$ | ${ }^{7} 2$ | 0 |

10. Mutual Replacement

| $22^{2} 2^{3} 5{ }^{4} 77^{5} 3$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 5 | 3 | 6 | 1 | 9 |
| 7 | 6 | 3 | 6 |  |

12. Unusual Friends

| 15 | 5 | 1 | 5 | 3 |
| ---: | :---: | :---: | :---: | :---: |
| 2 | 3 | 0 | 0 | 3 |
| 58 | 5 | 5 | 6 | 1 |

14. Singles and Doubles

|  | $6{ }^{3}$ | ${ }^{3} 7^{4} 1$ | $1{ }^{5} 2$ |
| :---: | :---: | :---: | :---: |
| 8 | 6 | 71 | 13 |
| 4 | $9{ }^{8}$ | 5 | 03 |

## Useful Numbers

## Squares

| 16 | 25 | 36 | 49 | 64 | 81 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | 121 | 144 | 169 | 196 | 225 |
| 256 | 289 | 324 | 361 | 400 | 441 |
| 484 | 529 | 576 | 625 | 676 | 729 |
| 784 | 841 | 900 | 961 |  |  |

## Triangulars

| 10 | 15 | 21 | 28 | 36 | 45 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 55 | 66 | 78 | 91 |  |  |
| 105 | 120 | 136 | 153 | 171 | 190 |
| 210 | 231 | 253 | 276 | 300 | 325 |
| 351 | 378 | 406 | 435 | 465 | 496 |
| 528 | 561 | 595 | 630 | 666 | 703 |
| 741 | 780 | 820 | 861 | 903 | 946 |
| 990 |  |  |  |  |  |

Fibonacci

| 13 | 21 | 34 | 55 | 89 |
| :---: | :---: | :---: | :---: | :---: |
| 144 | 233 | 377 | 610 | 987 |

## Primes

| 11 | 13 | 17 | 19 | 23 | 29 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 31 | 37 | 41 | 43 | 47 | 53 |
| 59 | 61 | 67 | 71 | 73 | 79 |
| 83 | 89 | 97 |  |  |  |
| 101 | 103 | 107 | 109 | 113 | 127 |
| 131 | 137 | 139 | 149 | 151 | 157 |
| 163 | 167 | 173 | 179 | 181 | 191 |
| 193 | 197 | 199 | 211 | 223 | 227 |
| 229 | 233 | 239 | 241 | 251 | 257 |
| 263 | 269 | 271 | 277 | 281 | 283 |
| 293 | 307 | 311 | 313 | 317 | 331 |
| 337 | 347 | 349 | 353 | 359 | 367 |
| 373 | 379 | 383 | 389 | 397 | 401 |
| 409 | 419 | 421 | 431 | 433 | 439 |
| 443 | 449 | 457 | 461 | 463 | 467 |
| 479 | 487 | 491 | 499 | 503 | 509 |
| 521 | 523 | 541 | 547 | 557 | 563 |
| 569 | 571 | 577 | 587 | 593 | 599 |
| 601 | 607 | 613 | 617 | 619 | 631 |
| 641 | 643 | 647 | 653 | 659 | 661 |
| 673 | 677 | 683 | 691 | 701 | 709 |
| 719 | 727 | 733 | 739 | 743 | 751 |
| 757 | 761 | 769 | 773 | 787 | 797 |
| 809 | 811 | 821 | 823 | 827 | 829 |
| 839 | 853 | 857 | 859 | 863 | 877 |
| 881 | 883 | 887 | 907 | 911 | 919 |
| 929 | 937 | 941 | 947 | 953 | 967 |
| 971 | 977 | 983 | 991 | 997 |  |

